Some New Findings on the Self-similarity Property in Communications Networks and on Statistical End-to-end Delay Guarantee

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COMP-03-006

Release Date: February 21, 2003

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Abstract

Real-time communication requires performance guarantee from the underlying network. In order to analyse the network performance, we must find the traffic characterization in every server of the network. Due to the strong experimental evidence that network traffic is self-similar in nature, it is important to study the problems to see whether the superposition of two self- similar processes retains the property of self-similarity and whether the service of a server changes the self-similarity property of the input traffic. In this paper, we first discusses some definitions and superposition properties of self-similar processes. Then we gives a model of a single server with infinite buffer and prove that when the queue length has finite second-order moment, the input process being strong asymptotically second-order self-similar(sas-s) is equivalent to the output process also bearing the sas-s property. Given the method for determinating the worst case cell delay for an ATM switch with self- similar input traffic, we can determine the end-to-end delay for such real-time communications in an ATM network by summing the cell delay experienced by each of the ATM switch along each connection.

KEYWORDS:
Self-similar, End-to-end delay, ATM networks, short-range dependent.

This technical report also appeared as JNG05-01 MAY, 2001

香港浸會大學
HONG KONG BAPTIST UNIVERSITY

FACULTY OF SCIENCE
DEPARTMENT OF COMPUTER SCIENCE
TECHNICAL REPORT (JNG05-01) May, 2001

Title:

Some New Findings on the Self-similarity Property in Communications Networks and on Statistical End-to-end Delay Guarantee[1]

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Abstract:

Real-time communication requires performance guarantee from the underlying network. In order to analyse the network performance, we must find the traffic characterization in every server of the network. Due to the strong experimental evidence that network traffic is self-similar in nature, it is important to study the problems to see whether the superposition of two self-similar processes retains the property of self-similarity and whether the service of a server changes the self-similarity property of the input traffic. In this paper, we first discusses some definitions and superposition properties of self-similar processes. Then we gives a model of a single server with infinite buffer and prove that when the queue length has finite second-order moment, the input process being strong asymptotically second-order self-similar(sas-s) is equivalent to the output process also bearing the sas-s property. Given the method for determinating the worst case cell delay for an ATM switch with self- similar input traffic, we can determine the end-to-end delay for such real-time communications in an ATM network by summing the cell delay experienced by each of the ATM switch along each connection.

Keywords:

Self-similar, End-to-end delay, ATM networks, short-range dependent.

[1]The work reported was supported in part by the RGC Earmarked Research Grant under RGC/97-98/54, and by the FRG under FRG/97-98/II-76.
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1. Introduction

Real-time communication is a very important type of services for the integrated services networks. It requires performance guarantee(such as delay bound and loss rate) from the network. Such guarantee is possible only if we can analyze the network performance. To do this, we must model the traffic of the real-time communication in the networks.

Several empirical studies on the LAN, the VBR video traffic, the ISDN and other communication systems indicate that these traffic are self-similar in nature. For instance, Leland et al. [7] have demonstrated the self-similar nature of Ethernet traffic by a statistical analysis of the Ethernet traffic measurements at Bell-Core; Beran et al. [3] have demonstrated long-range dependence in samples of variable bit rate video traffic generated by a number of different codecs; and Paxson and Floyd [11] have concluded the presence of long-range dependence in TELNET and other wide area network traffic.

In the light of these strong experimental evidence it is important to examine in more details the possible implications that self-similar traffic may have on the design and performance of network systems. For example, real-time communications require the network to provide end-to-end delay guarantee. In order to analyze the delay of networks with self-similar traffic, we need to know the property of queueing systems with self-similar input traffic. In particular, there are two important questions need to study as shown in Figure 1: One is whether the superposition of self-similar processes retains the self-similarity properties; the other is whether a server mechanism will change the self-similarity nature of the traffic.

In [12], B. Tsybakov and N. D. Georganas point out that the superposition of two uncorrelated self-similar processes retain some asymptotically self-similarity property. S. Vamvakos and V. Anantharam [14] consider a special case of a leaky bucket system with long-range dependent input traffic, and prove that the output (departure) process is also long-range dependent.

In this paper, we focus on the superposition of self-similar processes and the property of the output process from a server with self-similar input. And the rest of the paper is organized as follows: Section 2 discusses the concepts of self-similar processes and provides some new kinds of self-similar definitions and their relationships. In section 3, we give the superposition property of two self-similar processes. We obtained the superposition of two uncorrelated stong asymptotically self-similar processes(or long-range dependent processes) is stong asymptotically self-similar (or long-range dependent). Since traffic arrival to a switch are multiplexed by many connections, this superposition property is very important for the analysis of queueing systems. We also discuss the superposition of two correlated self-similar processes, and the superposition of a short-range dependent process with a self-similar process.

Section 4 considers a model of a single server with infinite buffer, and prove that when the second-order moment of a queue length process is finite, the strong asymptotically second-order self-similarity(sas-s) properties of the input process and that of the output process are equivalent, which means that the self-similarity property will neither be removed nor added by any server mechanism with finite second-order moment of queue length. Thus, given the method for determinating the worst case cell delay for an ATM switch with self- similar input traffic, together with this proof, we can determine the end-to-end delay for such real-time communications in an ATM network by summing the cell delay experienced by each of the ATM switch along each connection.

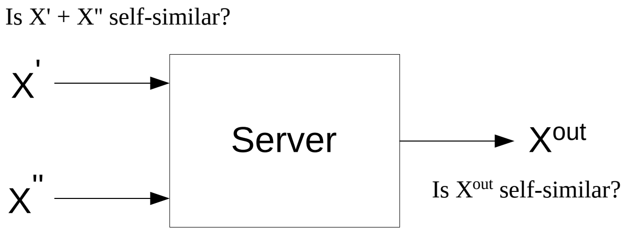

Figure 1. Self-similarity Properties.

2. Definitions of Self-similar

In this section, we first discusses some definitions of self-similar processes which are all defined based on a second-order-stationary real-number stochastic process.

We begin with the introduction of $X = (X_1, X_2, \ldots)$, a semi-infinite segment of a second-order-stationary real-number stochastic process of discrete argument (time) $t \in I_1 \triangleq \{1, 2, \ldots\}$ – the symbol $\triangleq$ means the equality by definition. Denote the mean and variance of X_t respectively by

$$\mu \triangleq EX_t < \infty \quad (1)$$

$$\sigma^2 \triangleq \mathrm{var}X_t = E(X_t - \mu)^2 < \infty \quad (2)$$

Denote the correlation coefficient and autocovariance of process X by

$$r(k) \triangleq \frac{E[(X_{t+k} - \mu)(X_t - \mu)]}{\sigma^2} \quad (3)$$

$$b(k) \triangleq \sigma^2 r(k) \qquad k \in I_0 \triangleq \{0, 1, 2, \ldots\} \quad (4)$$

Note that $r(0) = 1, b(0) = \sigma^2$, $r(k) = r(-k)$, and $b(k) = b(-k)$.

Definition 2.1 *A process X is called* **exactly second-order self-similar (es-s)***with Hurst parameter $H = 1 - (\beta/2)$ (see [2]), $0 < \beta < 1$, if its correlation coefficient is*

$$r(k) = g(k), \qquad k \in I_1 \quad (5)$$

where $g(k) \triangleq \frac{1}{2}[(k+1)^{2-\beta} - 2k^{2-\beta} + (k-1)^{2-\beta}],\ k \in I_1$

The function $g(k)$ can be written as

$$g(k) = \frac{1}{2}\delta^2(k^{2-\beta}) \quad (6)$$

where δ is the central difference operator

$$\delta(f(x)) = f(x + \frac{1}{2}) - f(x - \frac{1}{2}) \quad (7)$$

and δ^2 is the central second difference operator.

For the presentation of the next definitions, we need to introduce the averaged (over blocks of length m) process of X.

$$X^{(m)} = (X_1^{(m)}, X_2^{(m)}, \ldots) \tag{8}$$

$$X_t^{(m)} = \frac{1}{m}(X_{(t-1)m+1} + \ldots + X_{tm}),\ m, t \in I_1 \tag{9}$$

its variance

$$V_m \triangleq \mathrm{var} X_t^{(m)}. \tag{10}$$

and its correlation coefficient and autocovariance:

$$r_m(k) = \frac{E[(X_{t+k}^{(m)} - \mu)(X_t^{(m)} - \mu)]}{V_m}, \tag{11}$$

$$b_m(k) = V_m r_m(k) \tag{12}$$

Note that $b(k) \equiv b_1(k), \quad V_m = b_m(0)$.

The followings are the definitions of regular variation (Refer to the Appendix E of [13])

Definition 2.2 *A measurable function $f(x) > 0$ satisfying*

$$f(ux)/f(x) \to u^\rho, \quad x \to \infty \tag{13}$$

for each positive u, is called the **index ρ regularly varying function (rvf)**. *If $\rho = 0$, then rvf $f(x)$ is called the* **slowly varying function (svf)**. *If $f(x)$ is an index ρ rvf then $f(x) = L(x)x^\rho$ where $L(x)$ is a svf.*

The followings are the definitions of long-range dependence (l-rd) processes(see [2],[5]).

Definition 2.3 *A second-order stationary process X is called* **generalized long-range dependent process (gl-rd)***with Hurst parameter $H = 1 - (\beta/2)$, $0 < \beta < 1$, if its correlation coefficient satisfies*

$$r(k) \sim cL(k)k^{-\beta}, \quad k \to \infty \tag{14}$$

where $L(k)$ is a slowly varying function(svf), c is a constant, $0 < c < \infty$.

Especially, if

$$r(k) \sim ck^{-\beta}, \quad k \to \infty \tag{15}$$

X is called a **long-range dependent process (l-rd)**.

Definition 2.4 *A second-order-stationary process X is called* **asymptotically second-order self-similar (as-s)***with Hurst parameter $H = 1 - (\beta/2)$, $0 < \beta < 1$, if*

$$\lim_{m\to\infty} r_m(k) = g(k),\ k \in I_1 \tag{16}$$

Definition 2.5 *A second-order stationary process X is called* **generalized strong asymptotically second-order self-similar (gsas-s)***with Hurst parameter $H = 1 - (\beta/2)$, $0 < \beta < 1$, if the variance of $X^{(m)}$ satisfies*

$$V_m \sim cL(m)m^{-\beta}, \quad m \to \infty \tag{17}$$

where $L(m)$ is a slowly varying function(svf), c is a constant, $0 < c < \infty$.

Especially, if

$$V_m \sim cm^{-\beta}, \quad m \to \infty \tag{18}$$

X is called a **strong asymptotically second-order self-similar(sas-s)**.

According to the Definitions 2.1-2.5 above and the Theorem 2 of [13], we have the following relationships

Lemma 2.1

$$\begin{aligned} X \text{ is } es-s \Rightarrow \quad & X \text{ is } l-rd \quad \Rightarrow X \text{ is } gl-rd \\ \Rightarrow \quad & X \text{ is } gsas-s \quad \Rightarrow X \text{ is } as-s \\ X \text{ is } es-s \Rightarrow \quad & X \text{ is } l-rd \quad \Rightarrow X \text{ is } sas-s \\ \Rightarrow \quad & X \text{ is } gsas-s \quad \Rightarrow X \text{ is } as-s. \end{aligned}$$

that is,

$$\{es-s\} \subset \{l-rd\} \subset \{gl-rd\} \subset \{gsas-s\} \subset \{as-s\}$$
$$\{es-s\} \subset \{l-rd\} \subset \{sas-s\} \subset \{gsas-s\} \subset \{as-s\}$$

3. The Superposition of Self-similar Processes

In this section, we'll discuss superposition of self-similar processes which is of great importance for network performance evalution. Our main concern is under what conditions the superposition of self-similar streams will produce a self-similar stream. In [12], statement 7 and 8 point lead to the following results:

Lemma 3.1 *(1) If X' and X'' are such uncorrelated processes that $r(k) \sim c_1 k^{-\beta_1}, k \to \infty$ for X' and $r(k) \sim c_2 k^{-\beta_2}, k \to \infty$ for X'', where c_i and $\beta_i, i = 1, 2$ are constants, $0 < c_i < \infty, 0 < \beta_i < 1$, then $X' + X''$ is an asymptotically self-similar process with parameter $H = 1 - \beta/2$ where $\beta = \min(\beta_1, \beta_2)$.*

(2) Let the uncorrelated processes X' and X'' be exactly second-order self-similar, X' with H_1 and X'' with H_2. If $H_1 = H_2 = H$ then $X' + X''$ is exactly second-order self-similar with parameter H. If $H_1 \neq H_2$, then $X' + X''$ is not exactly second-order self-similar but is asymptotically second-order self-similar with $H = max(H_1, H_2)$.

We'll consider the superposition of two self-similar processes and the superposition of a self-similar process and a short-range dependent process. We obtain some very nice results about the property of merging self-simiar data treams.

Theorem 3.1 *(1) If X' and X'' are uncorrelated long-range dependent processes with Hurst parameters H_1 and H_2 respectively, then $X' + X''$ is a long-range dependent process with parameter $H = \max(H_1, H_2)$.*

(2) If X' and X'' are uncorrelated strong asymptotically self-similar processes with Hurst parameters H_1 and H_2 respectively, then $X' + X''$ is strong asymptotically self-similar with $H = max(H_1, H_2)$.

Proof: (1) X' and X'' are uncorrelated $l-rd$ processes, then for $k \to \infty$, we have

$$r'(k) \sim c_1 k^{-\beta_1}, \quad r''(k) \sim c_2 k^{-\beta_2}$$

where c_i and $\beta_i, i = 1, 2$ are constants, $0 < c_i < \infty, 0 < \beta_i < 1$, $H_1 = 1 - \beta_1/2$, $H_2 = 1 - \beta_2/2$.

Suppose $\beta_1 > \beta_2$, then $\beta = min(\beta_1, \beta_2) = \beta_1$.

Because $r(k)$, the correlation coefficient of $X' + X''$, satisfies

$$\begin{aligned} r(k) &= E[(X'_{t+k} + X''_{t+k} - \mu' - \mu'')(X'_t + X''_t - \mu' - \mu'')]/(\sigma'^2 + \sigma''^2) \\ &= E[(X'_{t+k} - \mu')(X'_t - \mu') + (X''_{t+k} - \mu'')(X''_t - \mu'')]/(\sigma'^2 + \sigma''^2) \\ &\quad + E[(X'_{t+k} - \mu')(X''_t - \mu'')(X''_{t+k} - \mu'')(X'_t - \mu')]/(\sigma'^2 + \sigma''^2) \\ &= E[(X'_{t+k} - \mu')(X'_t - \mu') + (X''_{t+k} - \mu'')(X''_t - \mu'')]/(\sigma'^2 + \sigma''^2) \\ &= r'(k)\sigma'^2/(\sigma'^2 + \sigma''^2) + r''(k)\sigma''^2/(\sigma'^2 + \sigma''^2) \end{aligned} \tag{19}$$

so

$$\begin{aligned} \lim_{k\to\infty} r(k)/k^{-\beta} &= \tfrac{\sigma'^2}{\sigma'^2+\sigma''^2} \lim_{k\to\infty} r'(k)/k^{-\beta} + \tfrac{\sigma''^2}{\sigma'^2+\sigma''^2} \lim_{k\to\infty} r''(k)/k^{-\beta} \\ &= \tfrac{\sigma'^2}{\sigma'^2+\sigma''^2} \lim_{k\to\infty} (c_1 k^{-\beta_1})/k^{-\beta} + \tfrac{\sigma''^2}{\sigma'^2+\sigma''^2} \lim_{k\to\infty} (c_2 k^{-\beta_2})/k^{-\beta} \\ &= \tfrac{c_1\sigma'^2}{\sigma'^2+\sigma''^2} + \tfrac{\sigma''^2}{\sigma'^2+\sigma''^2} \lim_{k\to\infty} c_2 k^{-(\beta_2-\beta_1)} \\ &= \tfrac{c_1\sigma'^2}{\sigma'^2+\sigma''^2} \end{aligned} \tag{20}$$

That means $r(k) \sim ck^{-\beta}$, $k \to \infty$. The same result is also obtained when $\beta_1 \le \beta_2$.

So, $X' + X''$ is $l-rd$ with $H = \max(H_1, H_2)$.

(2) X' and X'' are uncorrelated $sas-s$ processes, then for $m \to \infty$, we have

$$V'_m \sim c_1 m^{-\beta_1}, \quad V''_m \sim c_2 m^{-\beta_2}$$

and $H_1 = 1 - \beta_1/2$, $H_2 = 1 - \beta_2/2$.

Suppose $\beta_1 > \beta_2$, then $\beta = min(\beta_1, \beta_2) = \beta_1$.

Due to $V_m = \text{var}(X' + X'') = V'_m + V''_m$, so

$$\begin{aligned} &\lim_{m\to\infty} V_m/m^{-\beta} = \lim_{m\to\infty} (V'_m + V''_m)/m^{-\beta} \\ &= \lim_{m\to\infty} (c_1 m^{-\beta_1})/m^{-\beta} + \lim_{m\to\infty} (c_2 m^{-\beta_2})/m^{-\beta} \\ &= c_1 + \lim_{m\to\infty} c_2 m^{-(\beta_2-\beta_1)} = c_1 \end{aligned} \tag{21}$$

That means $V_m \sim cm^{-\beta}$, $m \to \infty$.

We obtained the same result when $\beta_1 \le \beta_2$. So, we have prove that $X' + X''$ is a $sas-s$ process with Hurst parameter $H = 1 - \min(\beta_1, \beta_2)/2 = \max(H_1, H_2)$.

Theorem 3.2 *(1) X' and X'' are long-range dependent processes that their correlation coefficients $r'(k) \sim c_1 k^{-\beta_1}$, $k \to \infty$ for X' and $r''(k) \sim c_2 k^{-\beta_2}$, $k \to \infty$ for X''. If there exist $\beta_3, \beta_4 (> \min(\beta_1, \beta_2))$, such that $|cov(X'_t, X''_{t+k})| \le c_3 k^{-\beta_3}$, $|cov(X'_{t+k}, X''_t)| \le c_4 k^{-\beta_4}$, $k \to \infty$, where c_i and $\beta_i (i = 1, 2, 3, 4)$ are constants, $0 < c_i < \infty$, $0 < \beta_i < 1$, then $X' + X''$ is a long-range dependent process with parameter $H = 1 - \beta/2$ where $\beta = \min(\beta_1, \beta_2)$.*

(2) X' and X'' are strong asymptotically self-similar processes with Hurst parameters H_1 and H_2 respectively. If there exists $\beta_3 > \min(\beta_1, \beta_2)$, such that $|cov(X_t'^{(m)}, X_{t+k}''^{(m)})| \le c_3 m^{-\beta_3}$, $m \to \infty$, where c_i and $\beta_i (i = 1, 2, 3)$ are constants, $0 < c_i < \infty$, $0 < \beta_i < 1$, $\beta_1 = 2(1 - H_1)$, $\beta_2 = 2(1 - H_2)$, then $X' + X''$ is strong asymptotically self-similar with $H = max(H_1, H_2)$.

Proof: (1) X' and X'' are $l - rd$ processes, then for $k \to \infty$, we have

$$r'(k) \sim c_1 k^{-\beta_1}, \quad r''(k) \sim c_2 k^{-\beta_2}$$

where c_i and $\beta_i, i = 1, 2$ are constants, $0 < c_i < \infty, 0 < \beta_i < 1$, $H_1 = 1 - \beta_1/2$, $H_2 = 1 - \beta_2/2$.

Suppose $\beta_1 > \beta_2$, then $\beta = min(\beta_1, \beta_2) = \beta_1$.

Because $r(k)$, the correlation coefficient of $X' + X''$, satisfies

$$\begin{aligned} r(k) &= E[(X'_{t+k} + X''_{t+k} - \mu' - \mu'')(X'_t + X''_t - \mu' - \mu'')]/(\sigma'^2 + \sigma''^2) \\ &= E[(X'_{t+k} - \mu')(X'_t - \mu') + (X''_{t+k} - \mu'')(X''_t - \mu'')]/(\sigma'^2 + \sigma''^2) \\ &\quad + E[(X'_{t+k} - \mu')(X''_t - \mu'')(X''_{t+k} - \mu'')(X'_t - \mu')]/(\sigma'^2 + \sigma''^2) \\ &= E[(X'_{t+k} - \mu')(X'_t - \mu') + (X''_{t+k} - \mu'')(X''_t - \mu'')]/(\sigma'^2 + \sigma''^2) \\ &\quad + [\text{cov}(X'_{t+k}, X''_t) + \text{cov}(X''_{t+k}, X'_t)]/(\sigma'^2 + \sigma''^2) \\ &= [r'(k)\sigma'^2 + r''(k)\sigma''^2]/(\sigma'^2 + \sigma''^2) \\ &\quad + [\text{cov}(X'_{t+k}, X''_t) + \text{cov}(X''_{t+k}, X'_t)]/(\sigma'^2 + \sigma''^2) \end{aligned} \tag{22}$$

and

$$\begin{aligned} &\lim_{k\to\infty}[|\text{cov}(X'_{t+k}, X''_t) + \text{cov}(X''_{t+k}, X'_t)|]/[k^{-\beta}(\sigma'^2 + \sigma''^2)] \\ &\le \lim_{k\to\infty}(c_3 k^{-\beta_3} + c_4 k^{-\beta_4})/[k^{-\beta}(\sigma'^2 + \sigma''^2)] \\ &= \lim_{k\to\infty}(c_3 k^{\beta-\beta_3} + c_4 k^{\beta-\beta_4})/(\sigma'^2 + \sigma''^2) = 0 \end{aligned} \tag{23}$$

So

$$\begin{aligned} \lim_{k\to\infty} r(k)/k^{-\beta} &= \tfrac{\sigma'^2}{\sigma'^2+\sigma''^2} \lim_{k\to\infty} r'(k)/k^{-\beta} + \tfrac{\sigma''^2}{\sigma'^2+\sigma''^2} \lim_{k\to\infty} r''(k)/k^{-\beta} \\ &\quad + \lim_{k\to\infty}[\text{cov}(X'_{t+k}, X''_t) + \text{cov}(X''_{t+k}, X'_t)]/[k^{-\beta}(\sigma'^2 + \sigma''^2)] \\ &= \tfrac{\sigma'^2}{\sigma'^2+\sigma''^2} \lim_{k\to\infty}(c_1 k^{-\beta_1})/k^{-\beta} + \tfrac{\sigma''^2}{\sigma'^2+\sigma''^2} \lim_{k\to\infty}(c_2 k^{-\beta_2})/k^{-\beta} \\ &= \tfrac{c_1\sigma'^2}{\sigma'^2+\sigma''^2} + \tfrac{\sigma''^2}{\sigma'^2+\sigma''^2} \lim_{k\to\infty} c_2 k^{-(\beta_2-\beta_1)} \\ &= \tfrac{c_1\sigma'^2}{\sigma'^2+\sigma''^2} \end{aligned} \tag{24}$$

That means $r(k) \sim ck^{-\beta}$, $k \to \infty$. The same result is obtained when $\beta_1 \le \beta_2$.

Hence, $X' + X''$ is $l - rd$ with $H = \max(H_1, H_2)$.

(2) X' and X'' are $sas - s$ processes, so, for $m \to \infty$, we have

$$V'_m \sim c_1 m^{-\beta_1}, \quad V''_m \sim c_2 m^{-\beta_2}$$

and $H_1 = 1 - \beta_1/2$, $H_2 = 1 - \beta_2/2$.

Suppose $\beta_1 > \beta_2$, then $\beta = min(\beta_1, \beta_2) = \beta_1$. V_m satisfies

$$\begin{aligned} V_m &= \text{cov}(X_t'^{(m)} + X_t''^{(m)}, X_t'^{(m)} + X_t''^{(m)}) \\ &= V'_m + V''_m + 2\text{cov}(X_t'^{(m)}, X_t''^{(m)}) \end{aligned} \tag{25}$$

then

$$\begin{aligned}
\lim_{m\to\infty} V_m/m^{-\beta} &= \lim_{m\to\infty} [V'_m + V''_m + 2\text{cov}(X_t'^{(m)}, X_t''^{(m)})]/m^{-\beta} \\
&= \lim_{m\to\infty} (c_1 m^{-\beta_1})/m^{-\beta} + \lim_{m\to\infty} (c_2 m^{-\beta_2})/m^{-\beta} \\
&\quad +2\lim_{m\to\infty}[\text{cov}(X_t'^{(m)}, X_t''^{(m)})]/m^{-\beta} \\
&= c_1 + \lim_{m\to\infty} c_2 m^{-(\beta_2-\beta_1)} + 2\lim_{m\to\infty}[\text{cov}(X_t'^{(m)}, X_t''^{(m)})]/m^{-\beta} \\
&= c_1
\end{aligned} \tag{26}$$

That means $V_m \sim cm^{-\beta},\ m \to \infty$.

We obtained the same result when $\beta_1 \le \beta_2$. So, we have prove that $X' + X''$ is a $sas - s$ process with Hurst parameter $H = 1 - \min(\beta_1, \beta_2)/2 = \max(H_1, H_2)$. QED

Now, we'll consider the superposition of a self-similar process and a short-range dependent process.

A process X is said to exhibit **short-range dependent** property if $\sum_{k=0}^{\infty} |r(k)| < \infty$;

A process X is said to exhibit **long-range dependent** property if $\sum_{k=0}^{\infty} |r(k)| = \infty$.

It is obious that all kinds of self-similar processes above have long-range dependent property. And the processes based on exponential distribution have short-range dependent property. There are some propositions about short-range dependent processes:

If $r(k) \sim ck^{-\beta}, 1 \le \beta < 2, k \to \infty$, then X is short-range dependent;

If $r(k) \sim cL(k)\rho^k, 0 < \rho < 1, k \to \infty$, then X is short-range dependent;

If $V_m \sim cm^{-1}, m \to \infty$, then X is short-range dependent.

Theorem 3.3 *(1) If X' and X'' are uncorrelated second-order stationary processes, X' is $l - rd$, $r'(k) \sim c_1 k^{-\beta_1}$, $k \to \infty$, $0 < \beta_1 < 1$, $0 < c_1 < \infty$, and X'' is short-range dependent, $r''(k) \sim c_2 k^{-\beta_2}$, $k \to \infty$, $1 \le \beta_2 < 2$, $0 < c_2 < \infty$, then $X' + X''$ is a long-range dependent process with parameter $H = 1 - \beta_1/2$.*

(2) If X' and X'' are uncorrelated second-order stationary processes, X' is $l - rd$, $r'(k) \sim c_1 k^{-\beta_1}$, $k \to \infty$, $0 < \beta_1 < 1$, $0 < c_1 < \infty$, and X'' is short-range dependent, $r''(k) \sim c_2 L(k)\rho^{-k}$, $k \to \infty$, $0 < \rho < 1$, $0 < c_2 < \infty$, then $X' + X''$ is a long-range dependent process with parameter $H = 1 - \beta_1/2$.

(3) If X' and X'' are uncorrelated second-order stationary processes, X' is $sas - s$, $V'_m \sim c_1 m^{-\beta_1}$, $m \to \infty$, $0 < \beta_1 < 1$, $0 < c_1 < \infty$, and X'' is short-range dependent, $V''_m \sim c_2 m^{-1}$, $m \to \infty$, $0 < c_2 < \infty$, then $X' + X''$ is a strong asympotically self-similar process with parameter $H = 1 - \beta_1/2$.

Proof: (1) Due to $\beta_2 > \beta_1$, $r'(k) \sim c_1 k^{-\beta_1}$, $\quad r''(k) \sim c_2 k^{-\beta_2}$, $k \to \infty$, and $r(k) = [\sigma'^2 r'(k) + \sigma''^2 r''(k)]/(\sigma'^2 + \sigma''^2)$, so

$$\begin{aligned}
\lim_{k\to\infty} r(k)/k^{-\beta_1} &= \tfrac{\sigma'^2}{\sigma'^2+\sigma''^2} \lim_{k\to\infty} r'(k)/k^{-\beta_1} + \tfrac{\sigma''^2}{\sigma'^2+\sigma''^2} \lim_{k\to\infty} r''(k)/k^{-\beta_1} \\
&= \tfrac{\sigma'^2}{\sigma'^2+\sigma''^2} \lim_{k\to\infty}(c_1 k^{-\beta_1})/k^{-\beta_1} + \tfrac{\sigma''^2}{\sigma'^2+\sigma''^2} \lim_{k\to\infty}(c_2 k^{-\beta_2})/k^{-\beta_1} \\
&= \tfrac{c_1\sigma'^2}{\sigma'^2+\sigma''^2} + \tfrac{\sigma''^2}{\sigma'^2+\sigma''^2} \lim_{k\to\infty} c_2 k^{-(\beta_2-\beta_1)} \\
&= \tfrac{c_1\sigma'^2}{\sigma'^2+\sigma''^2}
\end{aligned} \tag{27}$$

That means $r(k) \sim ck^{-\beta_1}$, $k \to \infty$, $X' + X''$ is $l - rd$ with $H = 1 - \beta_1/2$.

(2) The proof is similar to that of (1).

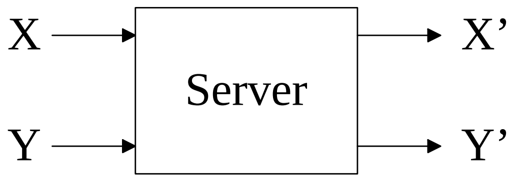

Figure 2. Self-similarity of the output process.

(3) X' and X'' are uncorrelated processes, $V'_m \sim c_1 m^{-\beta_1}$, $V''_m \sim c_2 m^{-1}$, $m \to \infty$, so

$$\begin{aligned} V_m &= \text{var}(X' + X'') = V'_m + V''_m \\ &\sim c_1 m^{-\beta_1} + c_2 m^{-1} \\ &= m^{-\beta_1}(c_1 + c_2 m^{\beta_1 - 1}) \\ &\sim c_1 m^{-\beta_1} \end{aligned} \tag{28}$$

That means, $X' + X''$ is a $sas - s$ process with Hurst parameter $H = 1 - \beta_1/2$. QED

4. The Self-similarity of the Output Process

Referring to Figure 2, we consider a single server queueing system with infinite buffer. For simplicity, suppose there are two classes of customers, and denote the input processes of the two classes of customers by X,and Y, such that X', and Y', respectively, are the corresponding output processes. (X, X') is the input-output processes pair that we are going to study, and (Y, Y') represents all other input-output processes pair. Denote $X^{(m)}, X'^{(m)}$ the averaged (over blocks of length m) processes of X, X' respectively.

we define $\hat{X}$, the queue length process corresponding to the arrival process X, as

$$\hat{X}_t = \text{the number of customer in queue from} X \text{ at time } t-$$

Assume that the average service rate is γ, and input processes X, Y are stationary and ergodic with arrival rate of α, β which satisfy the stability condition of $\alpha + \beta < \gamma$. According to the results in [1, 8, 14], we may consider a stationary regime in which the output process X', the queue length process, $\hat{X}$, is stationary and ergodic, and $\hat{X}_t$ has finite second order moment, (i.e.$E\hat{X}_t^2 < \infty$).

Let $r'_m(k), b'_m(k)$ be the correlation coefficient and the autocovariance of $X'^{(m)}$, respectively.

In the next section, we will study the problem: To see whether the output process X' is **sas-s**, when the input process is **sas-s**.

The following theorem is the major result of this paper, which states that the sas-s property of a network traffic is unchanged by the delay of any server satisfying the queueing length process which has a finite second order moment (see [6]).

Theorem 4.1 *In stationary regime, if $\hat{X}_t$ has a finite second order moment, then the followings are equivalent*

1. X is sas-s with Hurst parameter H (i.e. $V_m \sim cm^{-\beta}, \quad m \to \infty$).

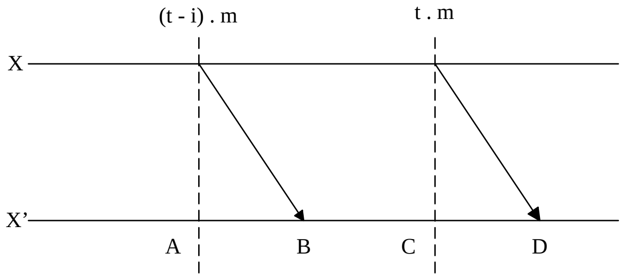

Figure 3. B before C.

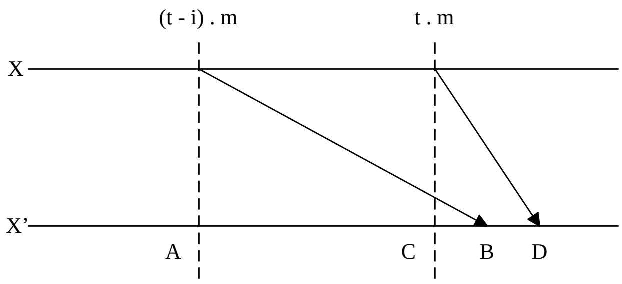

Figure 4. B after C.

2. X' is sas-s with Hurst parameter H (i.e. $V'_m \sim cm^{-\beta}, \quad m \to \infty$).

where $0 < \beta < 1$, c *is a constant.*

Moreover, X *being gsas-s with Hurst parameter* H *is equivalent to* X' *being gsas-s with Hurst parameter* H*, too.*

Proof: First of all, we show that

$$(X_t^{(m)} - X'^{(m)}_t) = (\hat{X}_{tm} - \hat{X}_{(t-1)m})/m \tag{29}$$

Denote A, and C at the time points $(t-1)m$, and tm, respectively. B, and D are the corresponding virtue output time point of an customer input at time $(t-1)m+,\ tm+$ under a FIFO regime. Note that it is uncertain that there is an customer input at this time, and the service is not necessary FIFO which will not effect the queue length and output process. We denote the numbers of output customer of input process X among the time intervals AB, AC, BD, CD by $\#(AB), \#(AC), \#(BD), \#(CD)$. Therefore, we have

$$\begin{aligned} X_t^{(m)} &= \#(BD)/m \\ X'^{(m)}_t &= \#(AC)/m \\ \hat{X}_{tm} &= \#(CD) \\ \hat{X}_{(t-1)m} &= \#(AB) \end{aligned} \tag{30}$$

then

$$\begin{aligned} |X_t^{(m)} - X'^{(m)}_t| &= |\#(BD) - \#(AC)|/m \\ (\hat{X}_{tm} - \hat{X}_{(t-1)m}) &= (\#(CD) - \#(AB)) \end{aligned} \tag{31}$$

Note that if $B < C$ (as in Figure 3) then

$$\begin{aligned} \#(BD) &= \#(CD) + \#(BC) \\ \#(AC) &= \#(AB) + \#(BC) \end{aligned} \tag{32}$$

On the other hand, if $B \geq C$ (as in Figure 4) then

$$\begin{aligned} \#(BD) &= \#(CD) - \#(CB) \\ \#(AC) &= \#(AB) - \#(CB) \end{aligned} \tag{33}$$

In both cases, it is true that

$$\#(BD) - \#(AC) \quad = \#(CD) - \#(AB) \tag{34}$$

that is

$$(X_t^{(m)} - X'^{(m)}_t) = (\hat{X}_{tm} - \hat{X}_{(t-1)m})/m \tag{35}$$

Next, we will prove an inequality

$$\begin{aligned} & \text{var}^{1/2}(X_t^{(m)}/V_m^{1/2}) - \text{var}^{1/2}(X_t^{(m)}/V_m^{1/2} - X'^{(m)}_t/V_m^{1/2}) \\ \leq \quad & \text{var}^{1/2}(X'^{(m)}_t/V_m^{1/2}) \\ \leq \quad & \text{var}^{1/2}(X_t^{(m)}/V_m^{1/2}) + \text{var}^{1/2}(X_t^{(m)}/V_m^{1/2} - X'^{(m)}_t/V_m^{1/2}) \end{aligned} \tag{36}$$

This inequality is equivalent to

$$|\text{var}^{1/2}(X_t^{(m)}/V_m^{1/2}) - \text{var}^{1/2}(X'^{(m)}_t/V_m^{1/2})| \leq \text{var}^{1/2}(X_t^{(m)}/V_m^{1/2} - X'^{(m)}_t/V_m^{1/2}) \tag{37}$$

and (37) are squared in two side

$$\begin{aligned} & \text{var}(X_t^{(m)}/V_m^{1/2}) + \text{var}(X'^{(m)}_t/V_m^{1/2}) - 2\text{var}^{1/2}(X_t^{(m)}/V_m^{1/2})\text{var}^{1/2}(X'^{(m)}_t/V_m^{1/2}) \\ & \leq \text{var}(X_t^{(m)}/V_m^{1/2} - X'^{(m)}_t/V_m^{1/2}) \end{aligned} \tag{38}$$

Using $\text{var}(X - Y) = \text{var}X + \text{var}Y - 2\text{cov}(X, Y)$, we can translate (38) into

$$\text{cov}(X_t^{(m)}/V_m^{1/2}, X'^{(m)}_t/V_m^{1/2}) \leq \text{var}^{1/2}(X_t^{(m)}/V_m^{1/2})\text{var}^{1/2}(X'^{(m)}_t/V_m^{1/2}) \tag{39}$$

This is right because

$$\frac{\text{cov}(X_t^{(m)}/V_m^{1/2}, X'^{(m)}_t/V_m^{1/2})}{\text{var}^{1/2}(X_t^{(m)}/V_m^{1/2})\text{var}^{1/2}(X'^{(m)}_t/V_m^{1/2})} \leq 1 \tag{40}$$

So we have proved (36).

At last, we prove that if $V_m \sim cm^{-\beta}, m \to \infty$, then $V'_m \sim cm^{-\beta}, m \to \infty$.

By (35),we obtain

$$(X_t^{(m)}/V_m^{1/2} - X'^{(m)}_t/V_m^{1/2}) = (\hat{X}_{tm} - \hat{X}_{(t-1)m})/mV_m^{1/2} \tag{41}$$

Since $V_m \sim cm^{-\beta}$, we have

$$\begin{aligned} mV_m^{1/2} \sim m\{cm^{-\beta}\}^{1/2} = \\ \{cm^{2-\beta}\}^{1/2} \to \infty, \quad m \to \infty. \end{aligned} \tag{42}$$

By stationary and $\hat{X}_t$ has a finite second order moment, we have

$$\text{var}(\hat{X}_{tm}) = \text{var}(\hat{X}_{(t-1)m}) = \text{var}(\hat{X}_1) < \infty \tag{43}$$

By (42) and (43),we can get the limit of the variance of right hand of (41)

$$\lim_{m\to\infty} \text{var}[(\hat{X}_{tm} - \hat{X}_{(t-1)m})/mV_m^{1/2}] = 0 \tag{44}$$

So, the variance of left hand side of (41) has limit

$$\lim_{m\to\infty} \text{var}(X_t^{(m)}/V_m^{1/2} - X'^{(m)}_t/V_m^{1/2}) = 0 \tag{45}$$

Since

$$\text{var}(X_t^{(m)}/V_m^{1/2}) = \text{var}(X_t^{(m)})/V_m = 1 \tag{46}$$

(36) can be written as

$$\begin{aligned} & 1 - \text{var}^{1/2}(X_t^{(m)}/V_m^{1/2} - X'^{(m)}_t/V_m^{1/2}) \\ \leq \quad & \text{var}^{1/2}(X'^{(m)}_t/V_m^{1/2}) \\ \leq \quad & 1 + \text{var}^{1/2}(X_t^{(m)}/V_m^{1/2} - X'^{(m)}_t/V_m^{1/2}) \end{aligned} \tag{47}$$

Let $m \to \infty$, we get that

$$\lim_{m\to\infty} \text{var}(X'^{(m)}_t/V_m^{1/2}) = 1 \tag{48}$$

By the definition of V'_m, we have

$$\lim_{m\to\infty} V'_m/V_m = 1 \tag{49}$$

that is

$$V'_m \sim cm^{-\beta} \tag{50}$$

Reversely and similarly, we can proof that if X' is sas-s, X is also sas-s. And using the same method, we can prove that X being $gsas - s$ is equivalent to X' being $gsas - s$.

QED

From this theorem, one can see that not only the sas-s properties of the input and output processes are equivalent, but their Hurst parameters $H = 1 - (\beta/2)$ are also the same.

For the condition of finite second-order moment of queue length, we could use the dynamic bandwidth allocation scheme (see [4]) or other methods to assure it. In fact, this condition is mathematically convenient, since the buffer sizes in practice are always finite, the queue length is bounded, and therefore, the condition is always satisfied.

5. End-to-end Delay

Theorem 4.1 can be applied to network traffic control for providing statistical guarantee for the end-to-end delay on switched networks. In [10], a statistical delay bound on an single ATM switch with self-similar Input Traffic has been given, according our result, the output process retains the same self- similarity property of the input process. Therefore, we can apply the same statistical delay bound on the subsequent ATM switches along this connection. The advantage is that we only need to calculate the Hurst parameter $H = 1 - (\beta/2)$ at the first ATM switch, and once in for all other switches along the same connection. Furthermore, given the method for determinating the worst case cell delay for an ATM switch with self-similar input traffic, with this proof, we can determine the end-to-end delay for such real-time communications in an ATM network by summing the cell delay experienced by each of the ATM switch along the connection.

A server S_i provide delay bound $\hat{d}_i$ guarantee with probability $1 - \delta_i$, means the delay of a cell in the connection satisfies:

$$P\{d_i \leq \hat{d}_i\} \leq 1 - \delta_i.$$

We have a result for the end-to-end delay bound:

Lemma 5.1 *If a connection go through N servers, and the server $S_i(i = 1, ..., N)$ provide delay bound $\hat{d}_i$ guarantee with probability $1 - \delta_i$, then $\sum_{i=1}^{N} \hat{d}_i$ is the end-to-end delay bound with guarantee probability $1 - \sum_{i=1}^{N} \delta_i$.*

Proof: For the given connection, the delay of a cell in server S_i is bounded by $\hat{d}_i$ with probability $1 - \delta_i$. Noted that d is the end-to-end delay. From the event $\{d \leq \sum_{i=1}^{N} \hat{d}_i\}$ we can imply

$$\begin{aligned} &\{d \leq \textstyle\sum_{i=1}^{N} \hat{d}_i\} \supset \{d_i \leq \hat{d}_i, i = 1, 2, ..., I\} \\ &= \cap_{i=1}^{N}\{d_i \leq \hat{d}_i\} = \overline{\cup_{i=1}^{I}\{d_i > \hat{d}_i\}} \end{aligned}$$

The end-to-end delay d satisfies:

$$\begin{aligned} P\{d \leq \textstyle\sum_{i=1}^{N} \hat{d}_i\} \quad &\geq P\{d_i \leq \hat{d}_i, i = 1, ..., I\} \\ &\geq 1 - \textstyle\sum_{i=1}^{N} P\{d_i \geq \hat{d}_i\} \\ &\geq 1 - \textstyle\sum_{i=1}^{N} \delta_i \end{aligned}$$

That means the end-to-end delay is bounded by $\sum_{i=1}^{N} \hat{d}_i$ with a guarantee probability of $1 - \sum_{i=1}^{N} \delta_i$.

6. Conclusion

In this paper, we have studied the performance guarantee of real-time communications from the network with self-similar traffic. The performance analysis require to know the property of traffic superposition. Because the output traffic of a switch is the input of the next switch, so we must study the self-similarity properties of output process.

Our results can be applied to network traffic control for statistical end-to-end delay guarantee on the networks. Given the method for determinating the worst case cell delay for an ATM switch with self-similar input traffic, we can determine the end-to-end delay for such real-time communications in an ATM network by summing the cell delay experienced by each of the ATM switch in the connection.

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